A Phase Estimator for Complex Signals using Evolutionary Algorithms

Jasdeep S Dhanoa, Evan J Hughes, *Member, IEEE* and Richard F Ormondroyd, *Member, IEEE*

***Abstract*—This paper presents a new method of phase estimation that is based on Evolutionary Algorithms. Unlike traditional phase estimators, this method does not require detailed *a priori* knowledge of the signal. The estimator operates by generating a model of the signal and optimising the model parameters including phase, to provide the best match in the frequency domain. The results show that the estimation process is capable of providing a good phase estimate of the signal, even in significant noise and for a wide range of waveforms.**

***Index Terms*—Phase estimation, Evolutionary Algorithms**

I. Introduction

PHASE estimation of signals is of great importance in applications such as interference cancellation, coherent communication over time-varying channels and direction of arrival estimation. The problem being addressed in this paper is the estimation of the initial phase of a complex signal for use in coherent systems. Djuric and Kay [1] have proposed several closely related methods that could be used for phase estimation of generalised chirps, that are described by higher order polynomials, with limited *a priori* knowledge about the signal characteristics. While these methods perform exceptionally well under conditions of high Signal-to-Noise Ratios (SNRs), for SNRs less than 8dB the performance of these methods degrades rapidly.

Sugahara et al. [2] proposed a combination of linear prediction and analysis by synthesis methods for the parameterisations of acoustic signals. This method is based on obtaining an approximate initial parameter estimate from the limited data samples available. Thereafter longer duration data are synthesised from this initial estimate and this synthesised signal is used for estimating the phase. The results show the performance of this method when no noise is present and its performance in noise has not been quantified.

Some methods based on expectation maximisation (EM) have been proposed for use in applications such as communications [3]. The EM method of waveform detection and phase estimation, in fact, corresponds to carrying out a maximum likelihood phase estimation using an iterative method. The results presented for the EM method show the performance under conditions when only the phase is unknown. In contrast, for the new method considered here, it is assumed that the *a priori* knowledge about the signal characteristics is limited and the number of unknown parameters, including the phase, is large.

This paper describes a new method for phase estimation of signals based on evolutionary algorithms (EA) that does not need accurate *a priori* knowledge of the signal parameters. This enables it to estimate the phase of any signal within a certain general description (such as non-linear chirps). This new method provides accurate estimates of the phase. Unlike many other methods whose performance quickly degrades in noise, the new estimation method degrades predictably with increasing noise levels.

Department of Aerospace, Power and Sensors, Cranfield University, Royal Military College of Science, Shrivenham, SN6 8LA, United Kingdom +44 1793785260 email j.s.dhanoa@rmcs.cranfield.ac.uk

II. The Concept

When there is limited *a priori* knowledge about the variables that characterise the received signal, accurate phase estimation is possible if the signal parameters are estimated along with the phase. It is clear that accurate estimation of the variables in the received signal would result in an accurate phase estimate.

Consider the case of phase estimation of linear chirps. It is assumed that limited *a priori* knowledge is that the chirp could lie anywhere within frequencies f_1 and f_2 and the chirp may be swept up or swept down in frequency, indeed it may even be a pure sinusoid.

In this case, any phase estimator would first need to estimate the chirp parameters i.e. the start and stop frequencies and only then would accurate phase estimation be possible. With such a large number of possible chirps, a maximum likelihood phase estimator would require a large number of templates to match against and this would be computationally intensive.

This new method is based on an algorithm that estimates the chirp parameters and the phase of the received signal concurrently. The method is capable of estimating phase with an accuracy of $\pm\frac{\pi}{20}$ radians with limited *a priori* knowledge of the signal parameters at an SNR of 10dB. The simultaneous estimation of the signal parameters and it's phase estimation is carried out by using an EA. The non-linear search process of the EA makes it suitable for this optimisation problem where the search space can be quite large and multi-modal with a number of local minimum.

III. Implementation using Evolutionary Algorithms

Evolutionary Algorithms are optimisation procedures which operate over a number of cycles (generations) and are designed to mimic the natural selection process through evolution and survival of the fittest [4]. A *population* of M possible solutions is maintained by the algorithm. Each potential solution is represented by one *chromosome*. This is the genetic description of the solution and may be broken into n sections called *genes*. Each gene represents a single parameter in the problem. The chromosome could be represented as vector $\overrightarrow{P}$ where the elements of the vector are the genes. Each trial solution forms a single point in the parameter space.

The three simple operations found in nature: natural selection, mating and mutation are used to generate new chromosomes and therefore new potential solutions. Each chromosome is evaluated at every generation using an *objective function* that is able to distinguish good solutions from bad ones and the chromosomes performance is assigned a score. With each new generation, some of the old chromosomes are removed to make room for the new improved offspring. Despite being very simple to code, requiring no directional or derivative information from the objective function and being capable of handling a large number of parameters simultaneously, evolutionary algorithms can achieve excellent results.

While there are various optimisation techniques available within Evolutionary Algorithms, we have found the Differential Evolution algorithm (DE) [5] to be particularly suitable for this application. The convergence properties of the DE algorithm appear to be more consistent than some other Evolutionary Algorithms such as Evolutionary Strategies and Evolutionary Programming [6] when applied to this problem.

A. *Differential Evolution*

Differential Evolution is an evolutionary technique that uses mutations that are related to the current spatial distribution of the population. The algorithm generates new chromosomes by adding the weighted difference between two chromosomes to a third chromosome. At each generation, for each member of the parent population, a new chromosome is generated. Elements of this new chromosome are then crossed with the parent chromosome to generate the child chromosome. The child chromosome is evaluated using the objective function and if it has a better objective value than the parent, the child chromosome replaces the parent. The size and direction of the difference between any pair of chromosomes is determined by the overall spread of the current population. Thus the DE algorithm self adapts to the fitness landscape, reducing the size of the mutations automatically as the search converges. In this way, no separate probability distribution has to be used for mutation which makes the scheme completely self-organising.

The trial chromosome $\vec{P}_t$ may be described as in (1).

$$\vec{P}_t = F(\vec{P}_a - \vec{P}_b) + \vec{P}_c \tag{1}$$

Where chromosomes $\vec{P}_a$, $\vec{P}_b$ & $\vec{P}_c$ are chosen from the population without replacement and F is a scaling factor.

The crossover process is controlled by a crossover parameter C. The crossover region begins at a randomly chosen parameter in the chromosome and then a segment of length L genes is copied from $\vec{P}_t$ to the parent chromosome to create the child chromosome. If the segment is longer than the remaining length of the chromosome, the segment is wrapped to the beginning of the chromosome. The length L is chosen probabilistically and is given by (2).

$$P(L \geq v) = (C)^{v-1}, v > 0 \tag{2}$$

In general, the scaling parameter F and the crossover parameter C lie in the range $[0.5, 1]$. Small values of F mean that the population spread reduces faster and this is more likely to result in the algorithm converging quickly at a local minima. We have found that values of 0.9 for both F and C are suitable for this application. Within the population, each individual chromosome represents a possible solution to the estimation and the gene values within the chromosome are the chirp parameters themselves.

B. *Chromosome Structure*

Within the context of the problem of phase estimation of complex signals considered here, the received signal/waveform is characterised by a set of parameters. Each of these parameters correspond to a single gene within a chromosome. Therefore a single chromosome contains a set of genes corresponding to the set of parameters that describe the waveform of interest. For a general case of linear chirps the signal can be described by (3) & (4).

$$x(t) = A\cos(f_c(t) + \phi) \tag{3}$$
$$f_c(t) = mt + c \tag{4}$$

The parameters that are required to completely regenerate the waveform are; m, c and the phase (ϕ). Thus the chromosome in this case contains three genes which correspond to these parameters.

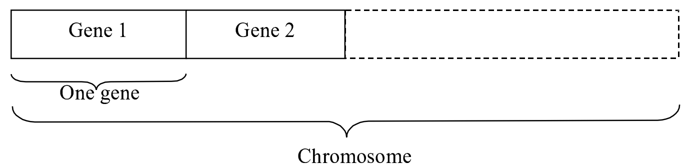

Fig. 1. Simplified block diagram of a chromosome structure

Figure 1 shows a schematic representation of such a chromosome. For a more general case of a waveform that is described by n parameters, the chromosome will consist of n genes.

The waveform that is characterised by each of these chromosomes represents a potential solution to the optimisation problem being considered here. An objective function is used to quantify the best match from within the population of chromosomes on the basis of mean square error, when comparing the total spectrum of the received signal with the locally generated model from the chromosome.

C. *Objective function*

The fitness of a particular chromosome in the population is based on: (a) regenerating the estimated chirped pulse from the genes, (b) obtaining the spectrum of this signal (*via* the FFT), (c) performing a least squares amplitude fit on both the real and imaginary components of the spectrum of the regenerated signal with the received signal and (d) comparing this amplitude scaled spectrum with the time averaged spectrum that was stored from the samples taken from the received pulse.

By using both the real and imaginary components of the frequency spectrum, it is not only possible to parameterise the parameters of the chirped signal but also to estimate its phase simultaneously.

The chromosome giving the least mean square error is chosen as the best match. The degree of match is quantified by using a mean square criterion given in (5).

$$E = \frac{\sum_{i=1}^{N}(x_i - y_i)^2}{N} \tag{5}$$

where x_i and y_i are the ith spectral components of the regenerated and received spectrum respectively, N is the total number of spectral components in the spectrum, E is the mean square error in the match between the two spectra being compared.

The problem addressed here is highly multi-modal; there are many local minima especially when the noise levels are high. Although optimisation using this EA is a non-linear least mean square process, unlike most least mean square methods it is not a gradient based optimiser. Consequently, although gradient based methods might converge to a local minima, the EA is much less likely to do so.

IV. Algorithm structure for an Evolutionary Algorithm based phase estimator

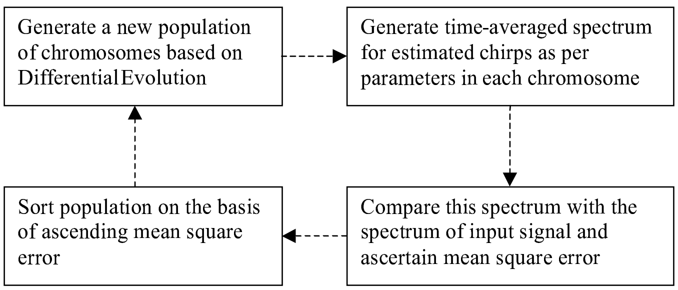

Fig. 2. Schematic block diagram of the algorithm for phase estimation and signal parameterisation

Figure 2 shows the schematic representation of the algorithm for phase estimation using Differential Evolution. The DE algorithm begins by generating an initial population of 350 random chromosomes with $F = 0.9, C = 0.9$, and the algorithm is run to convergence, which is generally less than 150 generations. For each generation, the DE algorithm evaluates each chromosome to find the best fit using a least mean square error approach. The chromosome giving the least mean square error is selected as the best fit and the others are arranged on the basis of ascending mean square error and from these a new population is generated. This new population retains some of the best chromosomes from the old population and chromosomes that would result in a poor solution are replaced with the new ones.

As can be seen, in addition to estimating the phase, the algorithm also parameterises the received pulse. Though the other signal parameters, like amplitude and it's time-frequency variations, are not being exploited in this paper, they may be of importance in applications such as spectral analysis.

V. Results

The results show the performance of the new method for different types of signal under varying SNRs. Each type of signal has different *a priori* knowledge.

The first case represents a non-linear chirp that may be found in acoustic signals. Many of the signal parameters are unknown and there is limited *a priori* knowledge.

The second case is a linear chirp applied to a high frequency electromagnetic wave. Here the original chirp parameters are known, however these parameters have changed due to an unknown Doppler shift.

A. Case I: Signal parameters unknown

For this set of results, it is assumed that the only *a priori* knowledge about the signal is that it is potentially a non-linear chirp that lies within frequencies $f_a = 200Hz$ and $f_b = 300Hz$ and with reference to (3), it's time-frequency variation can be represented by the general equation:

$$f_c(t) = ct^2 + bt + a \tag{6}$$

For the purpose of convenience, the coefficients c, b, a may be expressed as in (7).

$$\begin{aligned} c &= f_2 - f_1 - s \\ b &= s \\ a &= f_1 \end{aligned} \tag{7}$$

where f_1 and f_2 are the start and stop frequencies that lie somewhere within a wider frequency band f_a and f_b and, s is a parameter of non-linearity. This means that the search space involves four variables, three of which characterise the time-frequency variation of the signal and the fourth variable is the phase of the non-linear chirp, ϕ.

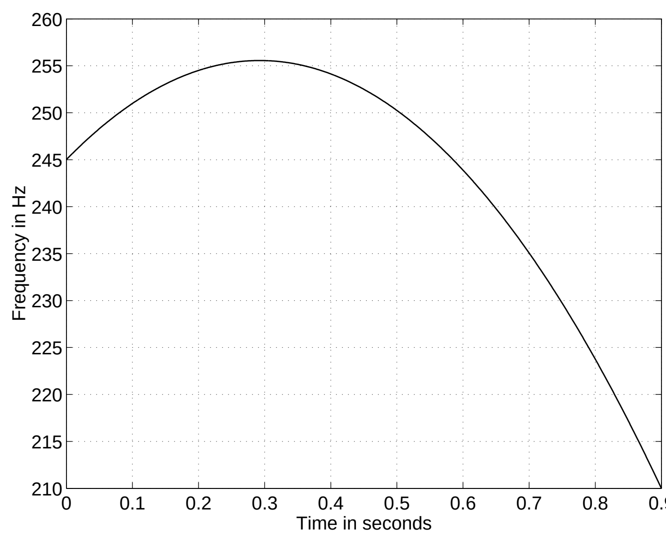

Fig. 3. Plot of the time-frequency variation of the test signal

To quantify the performance of the new method, a non-linear chirp test signal was generated. The time-frequency plot of the test signal is shown in Fig.3.

Figure 4 shows a section of the test signal at a SNR of 10dB, sampled at 1000 samples/s and Fig.5 shows it's corresponding spectrum.

Figure 6 shows the histogram of the errors in the estimate of the initial phase of this signal, in degrees, for 500 runs at an SNR of 10dB. The mean of the error is -0.488 degrees and the standard deviation is 3.520 degrees.

The test was then repeated at an SNR of -5dB, and the results are shown in Fig.7. This signal was also sampled at 1000 samples/s. Figure 8 shows it's corresponding spectrum.

Figure 9 shows the histogram of the errors in the phase estimation for 500 runs at a SNR of -5dB. The mean of the error

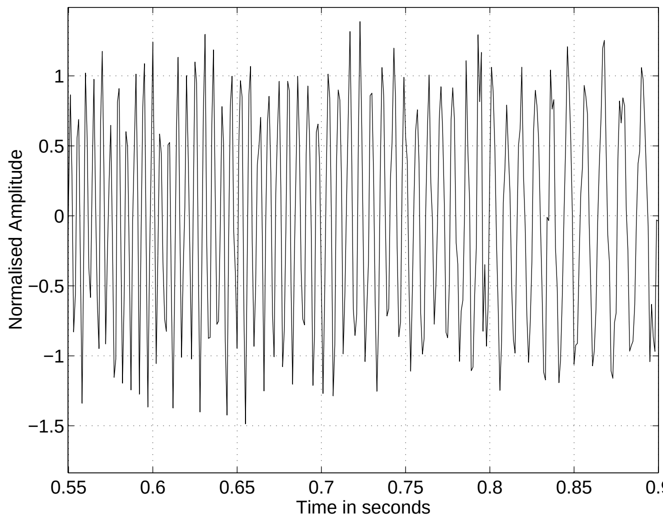

Fig. 4. Plot of the time-frequency variation of the test signal at SNR=10dB

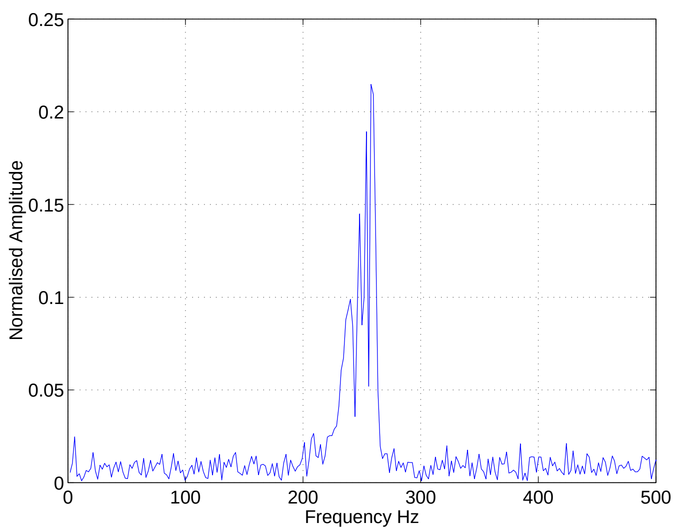

Fig. 5. Spectrum of the signal at SNR= 10dB

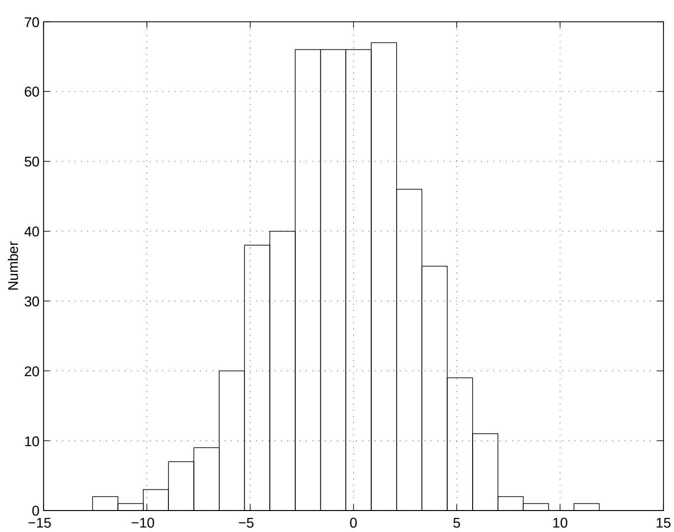

Fig. 6. Histogram showing the errors in phase estimate at SNR= 10dB

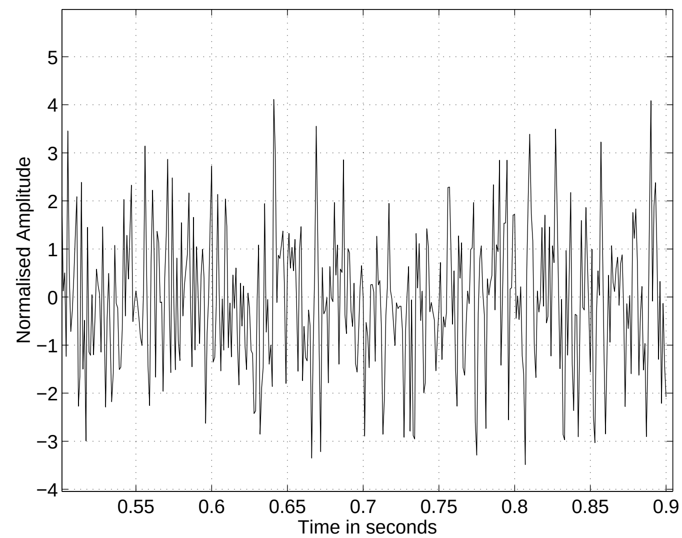

Fig. 7. Plot of the time-frequency variation of the test signal at SNR=-5dB

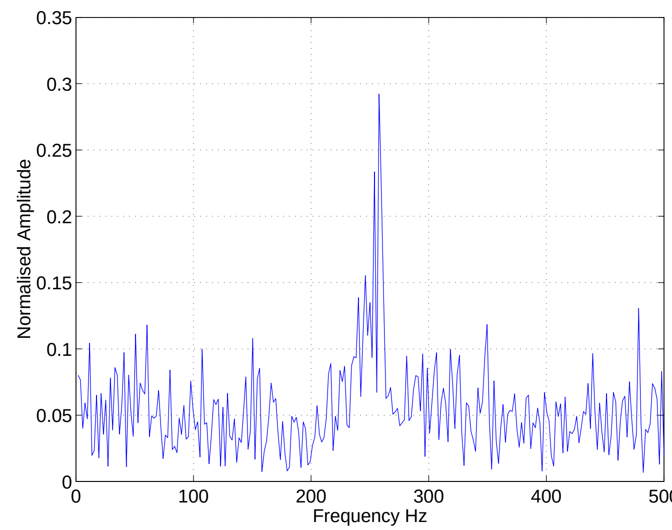

Fig. 8. Spectrum of the signal at SNR= -5dB

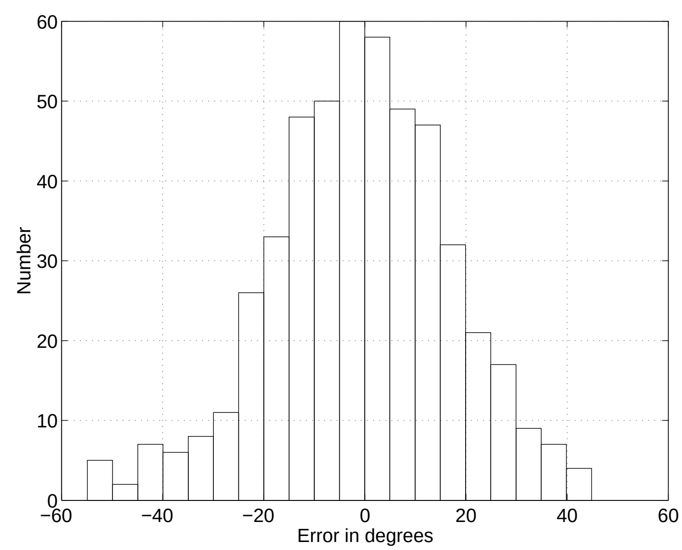

Fig. 9. Histogram showing the errors in phase estimate at SNR= -5dB

is -0.813 degrees and the standard deviation is 17.866 degrees. Note the difference in the *x* axis scales of Fig. 6 and Fig. 9.

These results show that even with very limited *a priori* knowledge about the signal parameters, the new method is able to estimate the initial phase of the signal despite the high levels of noise. It is also clear that the accuracy of the estimation improves with increasing SNR, as would be expected.

B. Case II: Signal parameters known a priori

For this case we consider an application where there is *a priori* knowledge that the received signal is a linearly chirped pulse of known bandwidth, however, there is an unknown Doppler shift of the signal due to platform motion. For this case, it would be essential to estimate the Doppler shifted frequency so as to obtain an accurate phase estimate.

TABLE I
LINEAR CHIRP PARAMETERS

Transmitted frequency	$35GHz$
Pulse duration	$10\mu s$
Chirp bandwidth	$15MHz$
SNR of received pulse	$-9dB$ *to* $21dB$
Range of expected Doppler velocities	± 6 *Mach*

Table I shows the chirp parameters of the transmitted signal that are known *a priori*. Although there is uncertainty in the frequency of the received chirp due to the unknown Doppler shift that it may have undergone, by knowing that it is a linear chirp and that its bandwidth is restricted, the search domain is reduced.

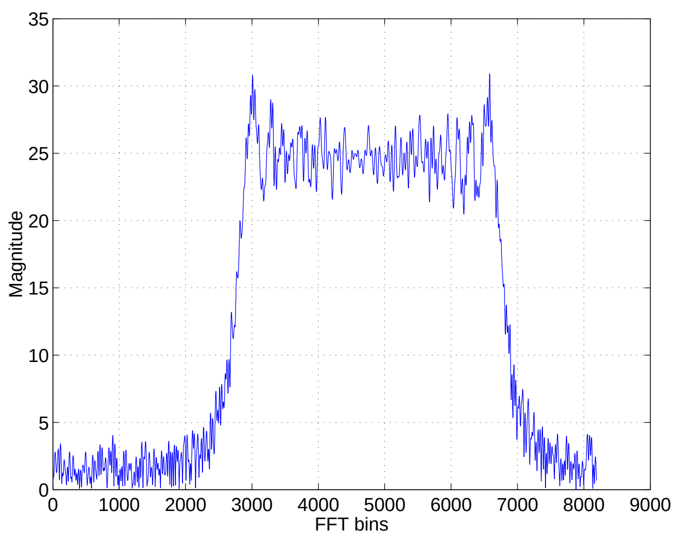

Fig. 10. Spectrum of test signal 2 at SNR= 21dB

Figure 10 shows the spectrum of the test signal at a SNR of 21dB, sampled at 30MHz. Figure 11 shows a histogram of the errors in the phase estimate at a SNR of 21dB for 1000 runs

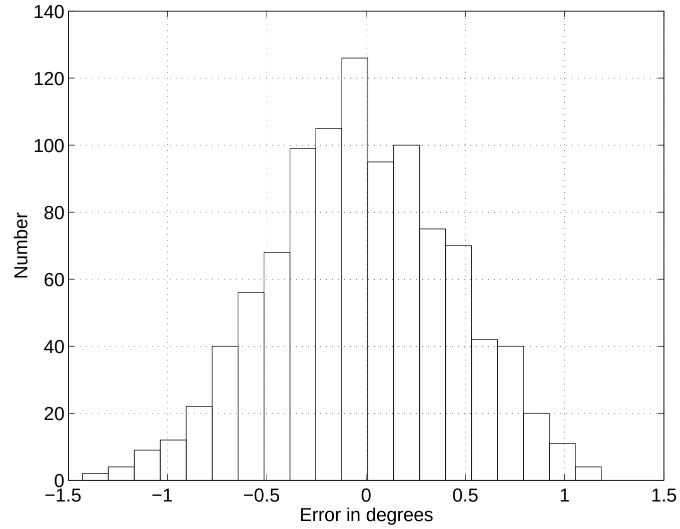

Fig. 11. Histogram showing error in phase measurement at SNR= 21dB

with an unknown Doppler shift. The mean of the error is -0.028 degrees and the standard deviation is 0.457 degrees.

A maximum likelihood phase estimator would require a large number of templates to estimate the phase of the signal with an accuracy of $\pm 1°$ for every possible Doppler shift.

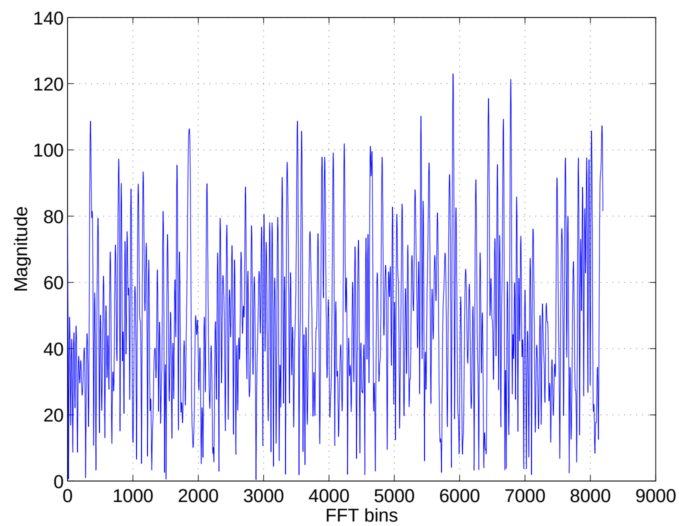

Fig. 12. Spectrum of test signal 2 at SNR= -9dB

Figure 12 shows the spectrum of the test signal at a SNR of -9dB, sampled at 30MHz. Figure 13 shows a histogram of the errors in the phase estimate at this SNR, also for for 1000 runs with an unknown Doppler shift. The mean of the error is -0.05 degrees and the standard deviation is 14.36 degrees. Again, please note the difference in the *x* axis scales of Fig. 11 & 12.

The performance of this new method was also tested at various other SNRs. Figure 14 shows the standard deviation in the errors of the phase estimate for varying SNRs over 1000 runs. The two lines depict the performance of the new method under different lengths of data. The plot shows the standard deviation of errors in degrees for the case when the duration of the signal is $10\mu s$ and $20\mu s$.

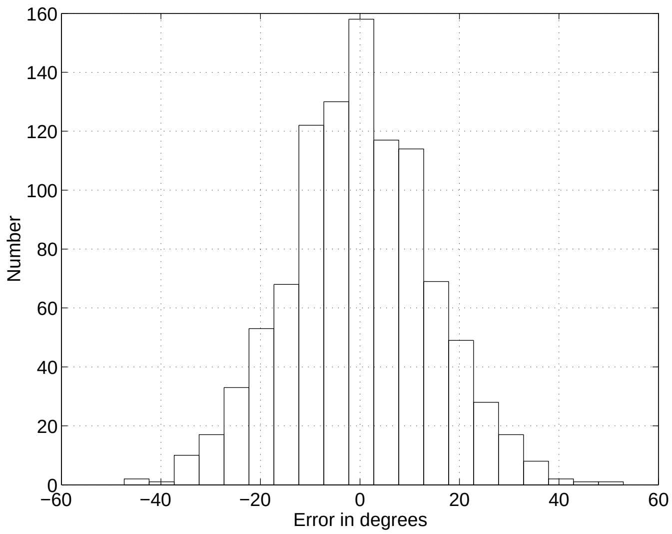

Fig. 13. Histogram showing error in phase measurement at SNR= -9dB

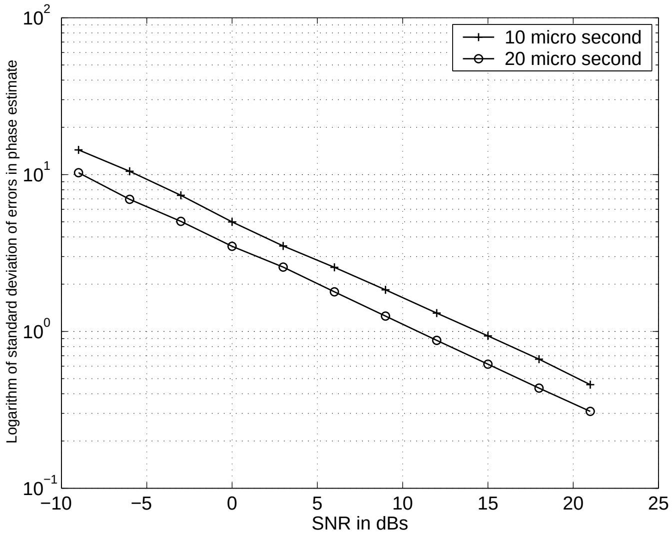

Fig. 14. Plot of logarithm of the standard deviation in the errors of phase estimate with varying SNR

It can be seen clearly that as the time duration of the received signal increases, the accuracy of the estimator improves. In addition, with decreasing noise levels, the errors in the phase estimate decrease predictably and there is no threshold of performance degradation unlike some methods. Even at a SNR of -9dB, the standard deviation of errors is less than $\pi/12$ radians.

VI. Conclusions

It is clear that despite having a limited knowledge of the actual signal parameters, the new method is able to estimate the phase of the received signal with high accuracy. As would be expected, the more information available about the signal, or if the time duration of the signal increases, the accuracy of the estimate of the initial phase improves.

The performance of the proposed method degrades predictably with noise as would be expected and even in the case of signal to noise ratios as low as -9dB the errors in the estimate are low. Being such a generalised phase estimator, it can be used in applications where the signal characteristics might be altered by the channel. One such application could be in the accurate phase estimation for a coherent communication system where the time varying channel could drastically alter the characteristics of the transmitted signal.

References

[1] P.M. Djuric and S.M. Kay, "Parameter estimation of chirp signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 12, pp. 2118–2126, Dec. 1990.

[2] Y. LI, K SUGAHARA, T. OSAKI, and R. KONISHI, "On the parameter estimation of frequency components," *5th International conference on, Signal Processing Proceedings,2000. WCCC-ICSP 2000.*, vol. 1, pp. 169–176, 2000.

[3] C.R. Nassar and M.R. Soleymani, "Joint sequence detection and phase estimation using the EM algorithm," *Conference proceedings, Electrical and Computer Engineering 1994.*, vol. 1, pp. 296–299, 1994.

[4] K. Deb, *Multi-objective Optimisation using Evolutionary Algorithms*, John Wiley and Sons, 2001.

[5] R. Storn and K. Price, "Differential Evolution- a simple and effective adaptive scheme for global optimization over continuous spaces," http://http.ICSI.Berkeley.edu/storn/code.html, last accessed Feb 2003.

[6] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley Publishing Company, Inc., 1989.