PERMITTIVITY MEASUREMENT USING A COAXIAL PROBE

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Abstract: **There are many applications that require knowledge of the complex permittivity of dielectric materials in the RF & microwave industry and for materials analysis. Many methods for the measurement of permittivity are available and the most suitable method is often driven by the frequency band and the nature of the dielectric materials in question. This paper presents a coaxial probe technique that interfaces with a vector network analyser to provide a convenient wideband measurement within the microwave band. It solves a thorough and complex model for the admittance at the probe end quickly and requires only a short-circuit reference in order to achieve simplicity of operation and accurate results. Some initial results using the coaxial probe are presented which confirm its accuracy.**

Introduction

There are many aspects of RF & microwave technology that demand knowledge of the complex permittivity of dielectric materials, such as the characterisation of semiconductors, radomes, absorbent materials, lens antennas and other structures. The propagation of RF signals and their interaction with the environment is key to an understanding of communications systems, radar, radiometric and medical imaging sensors and this requires dielectric characterisation of many natural and man-made materials such as stealthy structures, building materials, (sea) water, soils, vegetation and other biological tissues. Furthermore, dielectric data may be used in the analysis of materials such as the assessment of the structural integrity of composite structures [[1],[2]] and the water content of foods [[3]], timber, fuels and other chemicals. Dielectric data has the potential to be used in medicine to monitor the blood sugar content of diabetic patients [[4][5],[6]] and in the detection of tumours [[7],[8],[9]], melanomas and assessment of burns [[10]].

There are a host of techniques in use for the measurement of dielectric parameters such as complex permittivity. Specific measurement techniques are often better suited to certain material types (gases, liquids, soft or hard/crystalline solids), the expected dielectric behaviour (low-loss or high-loss materials), accuracy and bandwidth requirements than other techniques. Methods based on the perturbation of resonant structures yield highly accurate results but at one spot frequency whereas wideband techniques tend to lack the accuracy of spot frequency techniques. Some methods, e.g. based on free-space measurements, are ideally suited to the upper microwave and millimetric wave bands but do not scale conveniently to lower bands [[1]].

This paper offers a technical description of the technique developed by White Horse Radar (WHR) Limited’s CDP/0.5-40/1 coaxial dielectric probe for the measurement of dielectric properties in the microwave band. It is best suited to the measurement of complex permittivity across the centimetric wave band (500MHz to 40GHz) of liquids and soft or damp solids or hard solids having a smooth, flat surface that can be brought into good all-round contact with the probe.

The measurement is made by connecting the coaxial dielectric probe to a vector network analyser (VNA) to determine the admittance at the open surface of the probe when in contact with the material under test. The admittance result is then processed in accordance with a model to relate the probe admittance to the complex permittivity of the sample material. The quality of the solution is dependent on the efficacy of the model solved by the processing software, the calibration of the VNA and referencing of the probe and the quality of the contact between the probe and material under test. Unlike other coaxial probe kits on the market, the dielectric probe kit presented here uses just a short-circuit for its calibration; there is no need to calibrate the probe against a reference dielectric. This avoids errors associated with poorly characterised reference materials. The short-circuit reference is simple to apply and results in superior accuracy.

The remainder of this paper is organised as follows. Firstly, a description of some of the underlying theory of permittivity is offered. This is followed by a description of the model of an open coaxial probe. The next section describes the probe construction and measurement method. Some notes concerning the measurement bandwidth follow in the next section. There then follows a description of the data processing that is conducted to solve the admittance model of the probe. A short section is included to outline the pitfalls of using non-standard dielectric materials to reference probe measurements. Finally, results are presented to illustrate how the probes have already been used successfully.

Permittivity

Permittivity, ε, is a complex quantity which describes the ability of a dielectric to support an electric field and hence an electromagnetic signal. Thus:

$$\varepsilon = \varepsilon^{/} - j\varepsilon^{//} \qquad (1)$$

The real part of the permittivity, $\varepsilon^{/}$, is known as the dielectric constant, whereas the imaginary part of permittivity, $\varepsilon^{//}$, is the dielectric loss factor. The real and imaginary parts of permittivity may be depicted on a set of orthogonal axes as in

Figure below.

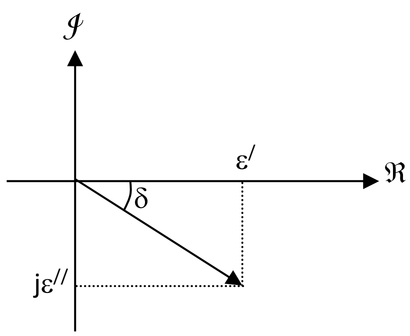

Figure 1: Complex Permittivity

A pure dielectric is loss-less and so $\varepsilon^{//}= 0$. In practice, most dielectrics are not ideal since they support some degree of conductivity and so $\varepsilon^{//}> 0$. It is worth noting that the imaginary part of permittivity is usually written as a negative quantity and so $\varepsilon^{//}> 0$ implies dielectric *loss*. Dielectric losses are also quantified using the loss tangent term i.e.

$$tan\,\delta = \frac{\varepsilon^{//}}{\varepsilon^{/}} = \frac{\sigma}{\omega\varepsilon^{/}} \tag{2}$$

where σ is the conductivity and ω is the angular frequency.

It is usual to refer to the *relative* permittivity as being the permittivity relative to that of free-space. Relative permittivity is defined as the factor by which the capacitance of a capacitor increases when the volume between and around its plates is filled with the dielectric as compared with free space. Therefore:

$$\varepsilon = \varepsilon_r\varepsilon_0 \tag{3}$$

where ε_r is the relative permittivity and ε_0 is the permittivity of free space and is equal to 8.854191 x 10^{-12} F/m. The relative permittivity can also be broken down into relative real and imaginary parts as:

$$\varepsilon_r = \varepsilon_r{}^{/} - j\varepsilon_r{}^{//} \tag{4}$$

and it is common to measure these *relative* parts of the complex permittivity.

Model of Admittance of an Open-Ended Coaxial Probe

The impedance, *Z*, or its inverse, admittance, *Y* (=1/*Z*), at the open end of a coaxial probe terminated in a dielectric medium is a function of its permittivity. For a semi-infinite half space of the medium with an infinite ground plane extending from the coaxial cable, as depicted in Figure 2, and assuming the presence of only the principal mode fields at the opening, the normalised aperture admittance is given by [[11],[12],[13]]:

$$Y = \frac{j\,k^2 Y_0}{\pi k_c\,ln(b/a)} \int_a^b \int_a^b \int_0^\pi cos(\varphi^{/}).\frac{exp(-jkr)}{r}\,d\varphi^{/}d\rho^{/}d\rho \tag{5}$$

where $$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r^{/}(1 - j\,tan\,\delta)} \tag{6}$$

and is the propagation constant of the medium terminating the probe and

$$k_c = \frac{2\pi f\sqrt{\varepsilon_c}}{c} \tag{7}$$

and is the propagation constant of the coaxial probe and

$$r = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi')} \qquad (8)$$

where a is the radius of the inner conductor of the coaxial probe,
b is the inner radius of the outer conductor of the coaxial probe,
c is the speed of light in free space (= 3 x 10^8 m/s),
Y_0 is the characteristic admittance of the coaxial probe line,
ε_r and $tan\delta$ refer to the dielectric sample terminating the probe,
ε_c is the permittivity of the coaxial probe dielectric,
μ_0 is the permeability of free space (= 4π x 10^{-7} H/m).

ρ, φ and z are cylindrical coordinates with the primed coordinates representing the source point and the unprimed coordinates representing field points.

The admittance given by (5) is normalised with respect to Y_0 which is the admittance of the coaxial probe (normally, Y_0 = 1/50 siemens).

The volume of the sample which couples to the measurement depends on the permittivity of the sample. In practice, an infinite ground plane and semi-infinite sample volume under the probe are not necessary; good accuracy is obtained for a ground plane extending a few centimetres from the probe centre and for samples of a few centimetres thickness.

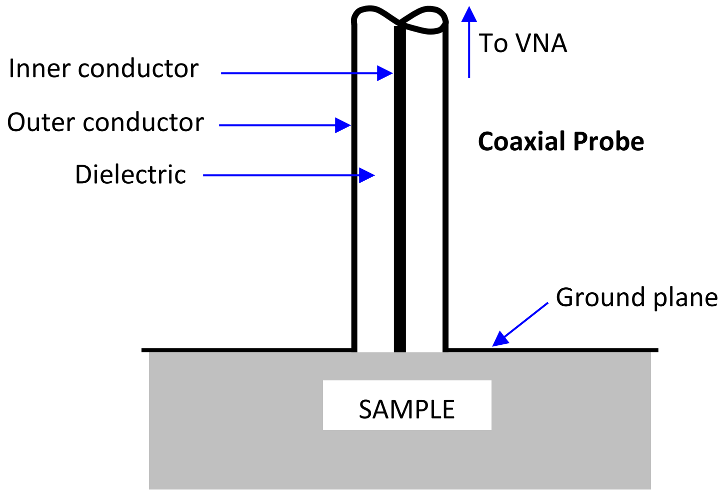

Figure 2: Coaxial Probe

The admittance at the probe tip may be computed from the reflection coefficient, S_{11}, at the probe tip since these quantities are related by:

$$Y = Y_0 \frac{(1-S_{11})}{(1+S_{11})} \qquad (9)$$

Both Y and S_{11} are complex quantities, although it is normal to refer to Y in its complex form, whereas it is normal to refer to S_{11} in polar coordinates i.e. having a magnitude of the reflection coefficient $|S_{11}|$ which is often expressed in a decibel (dB) ratio of $20\, log_{10}|S_{11}|$ dB and a phase angle of the reflection coefficient (in degrees).

Probe Construction & Measurement Method

The coaxial probe is manufactured from approximately 300mm of coaxial cable type 421-069 (0 to 40GHz) with a brass disc "ground" plate at one end of 40mm diameter and a 2.9mm microwave connector fitted at the other end. The brass flange at its end is in good electrical contact with the outer conductor of the coaxial cable and extends the ground plane in the plane of the probe tip. This brass ground plane is soldered to the coaxial cable with a high-quality connection all-round the end of the probe. The under surface is polished flat such that the coaxial cable inner conductor, outer conductor and extended ground are all in the exact same plane as each other. The coaxial cable with brass ground plane in place should be stabilised to the ambient temperature before use.

The vector network analyser (VNA) port one is connected to the microwave connector at the end of the probe. The probe is best supported vertically and clamped lightly into position. This enables a sample to be introduced from below without disturbing the probe and its connection with the VNA and is necessary so as to avoid any amplitude or phase disturbances to the system. The cable type 421-069 has the following parameters: a = 0.4mm, b = 1.166mm, ε_c = 1.687 and Y_0 = 0.02 S (i.e. 50Ω cable).

The VNA is configured to make a one port S_{11} measurement between 500MHz and 40GHz using 401 frequency points. The VNA is should be set up for an IF bandwidth of 10Hz (the lowest possible) and for 128 video averages in order to minimise the effects of noise and so achieve high accuracy. A time gating function is applied to isolate the reflection from the probe tip and so reject other reflections which may arise at the connection between the probe and the VNA that would corrupt the reading. The measurement bandwidth affords a distance resolution of about 3.8mm (free-space) or 2.9mm (in a cable of relative dielectric constant = 1.687), which is more than adequate to resolve the reflection from the probe tip from that at the microwave connector. The frequency step size is:

= bandwidth/number of frequency intervals
= 39.5GHz/400 = 98.75MHz

and gives a maximum unambiguous time of 10.13ns which corresponds to a two-way distance of 1.52 metres in free-space, or 1.17 metres in a cable of relative dielectric constant = 1.687. This maximum unambiguous range far exceeds the length of the probe and so the reflection from the probe/sample interface is captured without any ambiguity. [An ambiguous reflection coinciding with that from the probe tip would have to arise from an object which has an electrical distance of 1.52 metres (or multiples of 1.52m) beyond the probe tip; it is highly unlikely that any such reflection could be detected, or, conversely, any such reflection would be so weak that it would not provide any noticeable effect on the measurement from the probe tip.]

A short-circuit reference must be provided by holding a flat brass plate in contact with the probe surface. The magnitude of the reflection coefficient must be normalised to 0dB and the phase of the reflection coefficient to 180°, since there is a 180° phase inversion of the voltage at a short-circuit. This requires the input of a 180° phase offset during the reference run against the short-circuit. The reference data is stored in the VNA memory. For live measurements on samples, the 180° phase offset must now be removed. Measured data on samples is referenced to the short-circuit response using the VNA “data/memory” function. The VNA can display S_{11} (magnitude and phase) versus frequency data or can automatically compute equation (8) to display complex admittance on a Smith chart display, as desired. The user inputs the frequency and complex admittance into the data processing solver that computes the complex relative permittivity.

Bandwidth

The suggested bandwidth is 500MHz to 40GHz and is a suitable band for cable type 421-069 since the model assumptions start to break down and accuracy would be lost beyond this band. The volume of the sample which couples to the measurement depends on the permittivity of the sample. For lossy dielectrics a sample volume of several cubic centimetres is normally adequate.

At lower frequencies dielectrics tend to become less lossy and hence the fields around the end of the probe extend deeper into the sample under test. The volume of sample which couples to the measurement therefore tends to increase at lower frequencies and with samples of lower dielectric loss. The coaxial probe technique is not well suited to the accurate measurement of the dielectric loss factor of very low-loss dielectrics. The assumption that the sample under test occupies a semi-infinite half space starts to lose validity at lower frequencies because a sample of finite volume (especially thickness) allows the fields to interact with the sample container or material underneath the sample, such as the bench or whatever maybe supporting the sample. Furthermore, at lower frequencies the ground plane provided by the probe would have to be larger thus increasing the difficulty of manufacture or potentially sacrificing the good all-round contact between the probe and sample surface, particularly for solid dielectrics. The sample surface and the probe ground plane must be machined to a flat surface very accurately. Large samples would be required at lower frequencies, especially for low-loss dielectrics. The effects of large sample volumes coupling to the measurement are limited by the time gating function of the VNA, since the time gates isolate the reflection from the vicinity of the probe tip. More distant reflections (e.g. from supporting equipment, container walls and surfaces on the opposite side of samples) are rejected since they fall outside the time gate.

A simple test to verify whether the measurement is influenced by the limited sample thickness or materials beyond the sample under test is to hold a metallic layer, such as the short-circuit plate or copper tape, under the sample opposite the probe end. If the reading changes when the metallic layer is introduced, then the probe can sense the sample/air interface. The sample is too thin for a valid measurement. As a rough guide, the sample thickness should be at least 1cm, although low-loss dielectrics and /or low frequency measurements may require a thicker sample and highly lossy samples, and/or high frequency measurements could tolerate a thinner sample.

The model assumes the primary mode only at the probe tip. As one moves to higher frequencies, higher modes become possible and play an increasingly strong role. The cable type is limited to 40GHz for this very reason. In order to avoid the possibility of higher modes, an upper frequency limit of 40GHz is a sensible precaution. More complex models do exist that cater for higher modes, but these lead to far greater computational times. The upper frequency limit could be extended more realistically using an alternative cable type, but this requires solutions to the model for a different set of model parameters. In addition, coaxial probes operating well into the millimetre wave band require a high degree of precision in manufacture and use, leading to increased costs and/or less accurate results.

Data Processing

Solving the model of the probe described by equations (5) to (8) is problematic on two counts. First, it is not possible to invert these equations to find a closed form solution for ε_r given a measurement of Y. Secondly, the solution to the triple integral of (5) is complicated by the singularity which occurs when $\rho = \rho^{/}$ and $\varphi^{/} = 0$. In practice, setting the lower-limit of $\phi^{/}$ to a suitably small positive value enables the volume integral of equation (5) to converge to an accurate solution. This technique yields experimental errors which are smaller than experimental repeatability but at the expense of processing time.

The slow processing time has been overcome by pre-calculating solutions to equations (5) to (8) for a series of values of frequency, ${\varepsilon_r}^{/}$ and ${\varepsilon_r}^{//}$ and constructing a data table of the results. A measured value of Y can then be related to ${\varepsilon_r}^{/}$ and ${\varepsilon_r}^{//}$ for the particular measurement frequency by reference to the data table. Measured data is input to a simple program which loads in the data table and interpolates the correct complex permittivity result. The model equations (5) to (8), have been solved for 401 frequency points in the range 500MHz to 40GHz and over a range of ${\varepsilon_r}^{/}$ from 1 to 200 and ${\varepsilon_r}^{//}$ from 0 to 100. This forms the extent of valid results which can be processed. This supports an accurate real time permittivity result to be declared across the band 500MHz to 40GHz.

Pitfalls of Non-Standard Dielectric Reference Materials

Several other probes are marketed that use a very simplistic model for the admittance of a coaxial probe as a function of the sample permittivity [[14],[15]]. The simplistic model is parameterised on the basis of a measurement against a reference material of known permittivity. The reference material commonly used is water, although other reference materials may be used. Unfortunately, water is not suitable as a reference material because its dielectric properties are not consistent, and they are dependent on temperature. This objection is raised routinely on research papers whose dielectric measurements are based on a water reference. The accuracy of the such probe measurements is therefore poor due to the simplicity of the model and its dubious calibration technique used; these practices undermine research based on this equipment. Lists of reference dielectric materials are published by the National Physical Laboratory (NPL) [[16]].

Results

Measurements on deionised water between 10 and 30GHz are given in Table 1 and are in good agreement with previously published data [[17],[18]]. It ought to be stressed, however, that there is considerable spread in the published permittivity data for water due to differences in purity and temperature.

Frequency [GHz]	10	15	20	25	30
Relative Permittivity, ε_r	59.1 – j47.9	33.7 – j43.7	22.3 – j36.6	16.6 – j30.6	14.3 – j26.2

Table 1: Permittivity of Deionised Water

The relative permittivity of kerosene aviation fuel has been measured at $\varepsilon_r = 2.13 - j0.0012$. This measurement was made at a mid-band frequency of 15GHz and is reasonably consistent within the frequency range 1 to 30GHz and is also consistent with published data at 3GHz [[17]].

A variety of soil samples have also been measured in support of ground penetrating radar trials. The results would also be relevant to synthetic aperture radars used for remote sensing applications. The results are given in Table 2 below and are quoted as the mean several samples.

Soil Type	Number of samples	1 GHz	3 GHz	4 GHz
Top Soil - dry	2	3.35 – j0.50	1.65 – j0.10	3.10 – j0.02
Top Soil – slightly damp	2	5.00 – j0.85	3.30 – j0.50	4.45 – j0.60
Top Soil - wet	4	38.5 – j7.0	33.5 – j7.3	32.0 – j7.8
Sand – dry	1	3.40 – j0.50	1.85 – j0.10	3.20 – j0.00
Sand – slightly damp	3	3.63 – j0.50	2.00 – j0.16	3.40 – j0.02
Sand – wet	4	13.3 – j1.75	10.8 – j1.75	11.8 – j2.0
Clay - wet	4	42.5 – j8.0	37.0 – j8.5	35.5 – j8.8

Table 2: Permittivity of Soils

Some of the results for the dry samples are surprisingly low and are most likely due to poor all-round contact between the probe tip and granular samples. The contact improves with moist samples and the measurements are more consistent as a result. These results are in good agreement with previously published data [[17]].

Error sources include the accuracy of the model given by equation (5), the accuracy to which the probe coaxial cable is manufactured (accuracy of the probe parameters), the accuracy of the VNA measurement of admittance, the accuracy of the short-circuit reference, the efficacy of the all-round contact between sample and probe and the accuracy of the interpolation process in the data processing. In many cases it is not possible to quantify each of these error sources individually. Additionally, there is no closed solution method to translate errors in admittance into corresponding errors in permittivity. The dominant sources of error are believed to be the accuracy of the short-circuit reference and sample placement, which depend on the skill and diligence of the user. These may be quantified by making repeat measurements. The accuracy in the data processing has been quantified as less than a 1% error in permittivity and in many cases is < 0.1%. For reasonable tolerances in probe manufacture, variation in the probe parameters results in errors in permittivity of 2.2%. Errors in the declared permittivity result due to the interpolation process within the data processing are up to 2% in $\varepsilon_r^{/}$ and 0.5% or

3x10^{-3}, whichever is the greater, in $\varepsilon_r^{//}$. The percentage error in $\varepsilon_r^{//}$ increases for lower loss dielectrics, higher frequencies and higher $\varepsilon_r^{/}$.

Conclusions

A coaxial probe offers a quick and easy means of making wideband complex permittivity measurements on a wide variety of liquid and solid dielectric materials in the microwave band. The accuracy of the measurements is largely dependent on the quality of the probe referencing and sample placement but is also dependent on the accuracy of the data processing and the model relating the permittivity of the test material to the probe admittance. Accurate performance from the VHF band up to a few tens of GHz is possible. This band is limited by the extent of the validity of the assumptions built into the probe model and reasonable sample sizes. More accurate models cannot be solved in real-time but the use of pre-calculated look-up tables and interpolation permit near real-time measurements to be made. Coaxial probe systems that solve a simplified model that is parameterised by measurements on reference materials offer real-time results but incur greater modelling errors. Furthermore, any errors in the assumed permittivity of reference dielectrics add to the error of subsequent measurements. More accurate results can be obtained when only a short-circuit reference is required.

The WHR CDP/0.5-40/1 coaxial dielectric probe measurement system applies an accurate admittance model and requires only a short-circuit reference. It has demonstrated highly accurate results over the band 500MHz to 40GHz.

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