Medium PRF Set Selection Using Evolutionary Algorithms

Philip G. Davies, Evan J. Hughes

Abstract—**This paper presents a new and novel method of selecting multiple Pulse Repetition Frequency (PRF) sets for use in Medium PRF pulsed-Doppler radars. Evolutionary algorithms are used to minimise the blind areas in the range/Doppler space. The evolutionary algorithm allows optimal solutions to be generated quickly, far faster than with exhaustive searches, and is fully automatic, unlike existing techniques. The evolved solutions compare very favourably against the results of both an exhaustive search and existing published PRF set selection methods. This evolutionary approach to generation of PRF sets is a major advance in medium PRF radar design.**

Keywords— **Medium PRF Radar, Pulsed-Doppler Radar, Evolutionary Algorithms.**

I. INTRODUCTION

PULSED-DOPPLER radars operating in the medium PRF regime have been widely used in the last 30 years. With medium PRF systems, target returns are ambiguous in both range and Doppler frequency. To resolve these ambiguities it is necessary to use a transmitted waveform containing multiple PRFs.

The selection of the set of PRFs used in such waveforms remains even now something of a black art, due to the large solution space and the interdependence of the variables. Even one of the most recent methods proposed [1] is only able to find locally optimal solutions for half of the PRF set, the other half must be specified. In the past, the major factor that determined which PRF set is chosen has been decodeability. Techniques such as the Chinese remainder theorem [2] have been used, but for efficient operation, many constraints must be applied to the PRF set. With the power of modern processors however, the 'brute force' approach of coincidence lists, where the returns in each of the PRFs are overlaid and a search is performed for overlapping detections, allows a much wider choice of PRF [3]. This removes many of the constraints and so blind zone performance can now be a factor driving PRF set selection.

Evolutionary Algorithms have been shown to be useful in solving this type of combinatorial problem for other engineering applications, and have been used in this application to find the *globally* optimal set of PRFs, with no prior choice for any part of the PRF set needed. This fully automated system opens the door to selecting the optimal PRF set on-line to suit the current scenario and is a major advance in medium PRF pulsed-Doppler radar.

Evolutionary algorithms [4] have been developed by applying Darwin's theories of evolution and survival of the fittest to solve engineering problems. Their use is becoming widespread in many engineering (and other) applications. Evolutionary algorithms are less prone to getting stuck on local optima and more likely to find a global optima than conventional search methods.

Section 2 describes the problems associated with medium PRF radar and how the different PRF sets are classified. Sections 3&4 describe the ideas behind evolutionary algorithms and the specific algorithm used. Section 5 describes the results of the optimisation process, and section 6 concludes.

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II. MEDIUM PRF RADAR

The prime motivation in using pulsed-Doppler radar is to enable targets to be discriminated from the very large clutter returns present when the antenna main beam and side-lobes are incident on the ground. Designers are forced to accept the complexities inherent in medium PRF waveforms to gain good all-aspect performance in this 'look-down' situation. Morris [5] covers pulsed-Doppler techniques in depth. A typical radar of this type is the British Blue Vixen radar as fitted to the Sea Harrier which came into service in the early 1990s [6].

In airborne systems, main lobe clutter is the overwhelming return from the ground in the antenna main beam. This return has a spread of Doppler frequencies caused by the different relative velocities of the ground in the main beam footprint and repeats in the Doppler frequency domain at intervals of the PRF, while existing at all ranges due to the range ambiguities. Side lobe clutter gives returns from the ground at all Doppler frequencies up to $\pm$the platform velocity due to the side lobes of the antenna. The strongest return is from directly below the platform and repeats in time at multiples of the pulse repetition interval. As the pulse length is a significant fraction of the PRI, the radar will be blind during the time when subsequent pulses are being transmitted, often called *eclipsing*. Eclipsed regions repeat in time at intervals of the PRI. These three effects overwhelm returns in certain bands of Doppler frequencies and ranges, causing so called blind regions where targets cannot be detected. The spread and locations of the blind regions vary with platform altitude, forward velocity, carrier frequency and antenna angle. For a tracking radar, ambiguities in either range or Doppler may be acceptable, as long as an established track can be used to resolve the ambiguities. In surveillance radar, both ambiguities have to be resolved.

To resolve the ambiguities in range and Doppler, multiple PRF waveforms are required, each PRF in the set having the blind regions appear at different locations within the range and Doppler space. Therefore, the problem is to select a set of PRFs such that all ranges and Doppler frequencies that the radar must cover fall in a clear region for at least the several PRFs that are needed to resolve the ambiguities in range and Doppler. In practice, to achieve satisfactory detection probabilities and ranges, three or more PRFs are required to be clear for any one target to ensure the ambiguities can be resolved. In a practical radar, a number of sets of PRF schedules maybe generated for different altitudes, velocities, and antenna scan positions. The set for the conditions closest to the current scenario is then chosen as the operating set.

The most common method of assessing the suitability of a PRF set is known as a blind zone map. It is a plot of all Doppler frequencies of interest against all ranges of interest, showing in its simplest form, all the areas of this range/Doppler space that are not covered by enough PRFs to detect the target reliably and are therefore said to be blind to it. Both Doppler frequency and range are effectively quantised in a real design resulting in a blind zone map that can be conveniently modelled as a matrix of cells. The objective of this study was to minimise the total number of blind cells. Figure 1 shows a blind zone map for 8 different PRFs, with the black areas indicating where there are less than three PRFs clear and therefore the radar is blind.

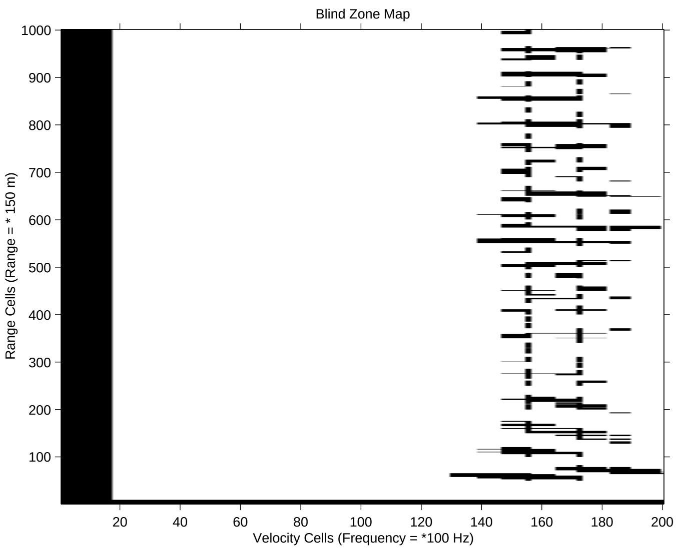

Fig. 1. Typical 3 of 8 PRF blind-zone map

The way that this is commonly achieved is known as an M of N scheme whereby a set of N PRFs are chosen with the aim that at least M different PRFs will be able to see a target anywhere in the range/Doppler space. Detections in these M PRFs can then be used to resolve the ambiguities [5]. A common choice for M and N is 3 of 8. With a 3 of 8 scheme, the PRFs can be calculated such that the ambiguous repetition of the main beam clutter notch is offset by $\frac{1}{4}$ of the notch width for each PRF, starting with the highest frequency and working down. While this ensures all Doppler frequencies of interest are covered by at least 4 PRFs, the coverage of these PRFs in range is found only by trial and error. It is not thought possible to calculate directly an optimum solution. The map in Fig. 1 is for a 3 of 8 scheme generated using the M of N method. The corresponding PRIs in terms of pulse widths are $[\ 50\ \ 53\ \ 55\ \ 58\ \ 61\ \ 64\ \ 68\ \ 72\ ]$.

For the purposes of this study, a model of an airborne pulsed-Doppler radar was used. The model parameters are based on the radar model used by Simpson [7], which although simplified in some ways, represents typical characteristics and parameters. See [8] for full details of the problem definition and implementation. The simplified model assumes that the blind zone map is normalised to centre the main-beam Doppler return around zero Hertz, and that the sidelobe return from the ground directly below the platform appears immidiately after the region blinded by eclipsing.

The radar parameters are based on an X-band airborne pulsed-Doppler medium PRF mode airborne fire control radar with the main parameters summarised in Table I. In the system used by Simpson, the Pulse Repetition Intervals (PRIs) were all an integer multiple of the pulse width (1 range cell). The upper and lower bounds on the available PRI set are often limited by system constraints [3]. Therefore with the radar parameters used, each PRF will be one of 47 discrete values (PRI of 50 pulse widths for highest PRF, to PRI of 96 pulse widths for lowest PRF). With a 3 of 8 scheme, the eight PRFs needed will result in a search space of 3.14×10^8 possible PRF sets.

TABLE I

SUMMARY OF RADAR CHARACTERISTICS

Parameter	Value
Frequency	10 GHz
Wavelength	3 cm
Min PRF freq.	10.4 kHz
Max PRF freq.	20.0 kHz
Pulse width	$1\mu S$
Range bin size	150 m

The blind range due to side lobe clutter and eclipsing is simulated by blinding a total of 11 cells at repetitions of the PRI in range (1 cell for eclipsing, 10 cells for sidelobe clutter). The main clutter notch is simulated by blinding 3.4 kHz (± 25.5m/s) at every repetition in frequency and corresponds to ± 17 cells at multiples of each PRF. The simulation was performed for 1000 150-metre range cells and 200 100Hz velocity cells giving a total region of 150km and 20kHz (=300 m/s). The unavoidably blind regions in range and Doppler beginning at cell zero are not included in the calculation of the blind area. The blind zone map is generated using a 1000×200 element array to record how many PRIs are blind in each cell. For each PRI, a corresponding array containing zeros for clear areas and ones for blind areas is added onto the main blind zone map array. Figure 2 shows a typical single PRI blind map. After the blind areas from all the PRIs have been accumulated, cells with more than 5 of the 8 PRIs blind are declared as being blind.

To allow range and Doppler ambiguities to be resolved properly, the PRF set chosen must be fully decodeable. To ensure decodeability, the lowest common multiple of any triplet of PRIs (and PRFs) must be outside of the space of interest in range (and Doppler). For example, PRIs of $50\mu S$, $60\mu S$ and $80\mu S$ have a lowest common multiple of $1200\mu S$ which is beyond the $1000\mu S$ range cell space of interest and therefore the triplet is decodeable in range. In an 8 PRI set, there are 56 possible triplets that all need to be decodeable in both range and Doppler for the complete set to be acceptable. In this paper, decodeability has not been included as a constraint on the evolutionary algorithm.

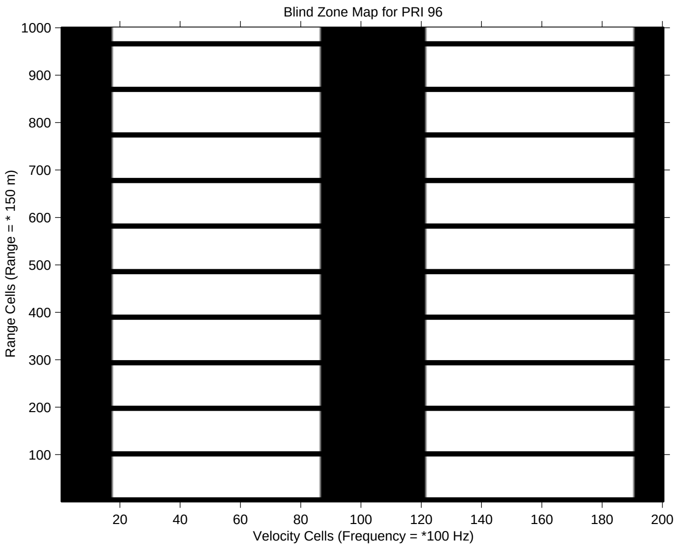

Fig. 2. Blind Map for PRI $96\mu S$

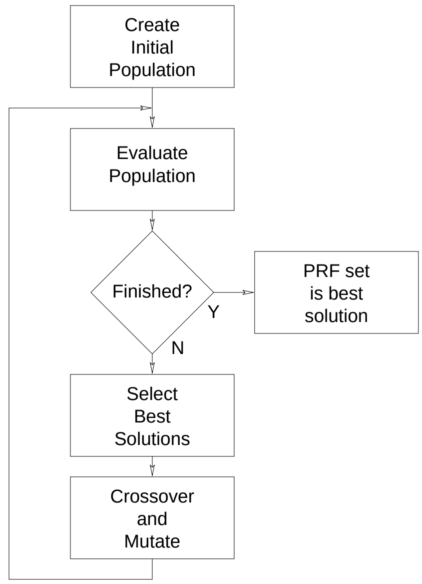

Fig. 3. Flow diagram of the evolutionary algorithm

III. Evolutionary Algorithms

Evolutionary Algorithms are designed to mimic the natural selection process through evolution and survival of the fittest. A *population* of P independent individuals is maintained by the algorithm, each individual representing a potential solution to the problem. Each individual has one *chromosome*. This is the evolutionary description of the solution and may be broken into n sections called *genes*. Each gene represents a single evolveable parameter in the problem.

The three simple operations found in nature, natural selection, mating and mutation are used to generate new chromosomes and therefore new potential solutions. Each individuals' chromosome is evaluated at every generation using an *objective function* that is able to distinguish good solutions from bad ones and to score their performance. In this paper, the objective function is the total blind area that is generated by the PRF set described by a chromosome passed to the objective function.

With each new generation, some of the old individuals die to make room for the new, improved offspring. Over several generations, the majority of the solutions represented by the individuals in the population will tend to lie around an optimal solution for a given environment. The exact rate at which the population converges to a single solution is determined by the nature of the problem and the structure of the evolutionary algorithm. The evolutionary process can be described by the flow diagram in Fig. 3.

When used to solve numerical optimisation problems, evolutionary algorithms that have a large population tend to search areas spread across the entire optimisation surface before converging on a maximum or minimum depending on the problem. Thus, despite being very simple to code, requiring no directional or derivative information from the objective function and being capable of handling large numbers of parameters simultaneously, evolutionary algorithms can achieve excellent results. A good introduction to evolutionary algorithms can be found in [9].

IV. Algorithm Construction

In the problem described in this paper, the set of parameters is the set of PRFs chosen. Thus the algorithm will strive to evolve the best set of PRFs that minimise the total blind area.

The evolutionary algorithm used encodes each of the PRF sets into a binary string (the chromosome). Each PRF of a set is generated from a corresponding 6 bit binary sequence from the chromosome. Unfortunately, the 47 possible choices for the PRF do not fit neatly into binary as a 6 bit representation has 64 possible values. Therefore if the decoded decimal equivalent of the binary representation is greater than 47, it is just set to be 47 before evaluating the objective function. With 8 PRFs and 6 bits per PRF, each chromosome contains 56 bits in total in order to describe one complete PRF set.

The number of blind cells is used to determine which solutions are the best. The lower the number of blind cells the better. The fitness value of each chromosome is assigned according to the rank position of the individual. The individual with the greatest objective value (least fit) is assigned a rank position of 1 and the best individual is assigned a rank position P. The ranked positions can then be used to generate probabilities that each of the chromosomes will survive to pass into the next generation. This allows the better solutions to generate more offspring. A technique called Stochastic Universal Sampling [4, Page 12] is used to select P individuals at random from the population for breeding, based on the previously derived probabilities of survival.

The individuals selected are paired up for breeding. Breeding pairs are chosen at random to ensure a good evolutionary mix. A method of generating new chromosomes is used called *Single-Point Crossover* [4, Page 12]. Two new offspring are generated from each pair of parents by swapping sections of their chromosomes. With a 70% chance of crossover, crossover operates by choosing randomly a position in one parent's chromosome, dividing it into two sections. The second parent's chromosome is then also divided at the same position. The chromosomes of the two parents' may now be represented as the strings of genes $[a_1\ a_2]$ and $[b_1\ b_2]$. The chromosomes of the two offspring may now be defined as $[a_1\ b_2]$ and $[b_1\ a_2]$. After crossover, mutation is applied by flipping bits with a probability of 0.05.

The new chromosomes are all evaluated in the objective function and the best 90% replace the worst 90% of the parent population. The algorithm is terminated after 50 generations and the best individual overall is recorded as the solution. The choice of 50 generations was arrived at empirically as most runs appeared to have converged satisfactorily by then. More generations could be used, but little benefit may be gained as the results found after 50 generations are likely to be sufficient (see section V). A population of either $P = 100$ or $P = 200$ individuals was used for the experiments, therefore giving 5000 and 10000 objective calculations respectively for each run of the algorithm.

V. Results

The experimental approach used was to generate solutions using an evolutionary algorithm and then to attempt to validate these results by conducting exhaustive searches of the solution space to find the actual global optimum and other near optimum solutions. This process was undertaken for both 3 of 5 and 5 of 7 schemes using otherwise equivalent parameters and constraints. This validation process then gave confidence that the results produced by the evolutionary algorithm for a practical example 3 of 8 scheme were likely to be optimum (or near optimum), as the 8 PRF exhaustive search was too large to be undertaken as part of this work. These solutions could then be compared against other published methods of set selection.

The exhaustive search was performed on the University's SGI Cray Origin 2000 supercomputer. For the 5 PRF problem, there are 1.53×10^6 possible combinations to try, taking a trivial time. However, using half the Cray's processors the 7 PRF problem has 6.29×10^7 combinations and takes about 3 days processing, rising to two and a half weeks for the 8 PRF problem's 3.14×10^8 combinations. Ten independent runs of the evolutionary algorithm on each of the three set sizes were used in order to try and estimate the probability of finding good solutions.

For the 5 PRF problem, the smaller population size of 100 individuals was used for 50 generations. The evolutionary approach could identify the global optimum solution in 8 out of the 10 runs, leading to an estimated 6250 evaluations needed to achieve the optimum solution, a massive saving over the exhaustive search. The number of blind cells though is quite high for a 3 of 5 system.

For the 7 PRF problem, a population size of 200 was used for 50 generations. The global optimum was identified in 2 out of the 10 runs, leading to an estimated 50000 evaluations needed to achieve the optimum solution, again a massive saving. The number of blind cells for the optimum solution of the 3 of 7 strategy is less than for the 3 of 8 system designed with the M of N method noted in section II.

The performance of the evolutionary algorithm on the 3 of 7 system suggests that for the 8 PRF problem, the evolutionary method may be capable of identifying the optimum solution in a very reasonable number of objective function evaluations, when compared to an exhaustive search. Using the published M of N method to calculate PRF sets, we could not identify the global optimum (based on better results found by the evolutionary method). The results were, however, reasonable. The evolutionary approach can identify solutions as good as the M of N approach very quickly indeed.

In the runs where the evolutionary algorithms did not find the global solution for the 5 and 7 PRF trials, the solutions they found were all very good. Table II shows the grouping of the results.

TABLE II
Summary of trial statistics

Result rank	5 PRF (%)	7 PRF (%)
Global optimum	80%	20%
In best 10	100%	20%
In best 100	100%	80%

For the 8 PRF problem, again a population size of 200 was used. This may lead to a sub-optimal solution though as the 7 PRF problem had only a 20% probability of finding the solution with the same algorithm. The total avoidable blind area (area excluding first main-beam clutter notch and first eclipse and sidelobe clutter) of the best solution found by the evolutionary algorithm is 305, 25 times less than that found by the M of N method. With a larger population and more generations, the probability of finding solutions as good as the best found so far will increase. The best found so far is shown in Fig. 4, compared to the M of N plot of Fig. 1. The corresponding PRIs in terms of pulse widths are [51 53 60 63 67 84 89 93].

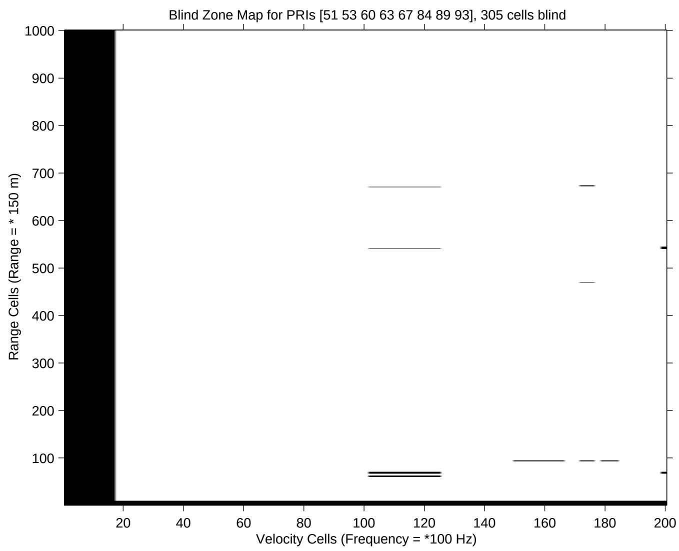

Fig. 4. Blind-zone map for 8 PRFs found by evolution

The blind zone performance in terms of the number of cells that are blind are shown in Fig. 5 for six different PRF selection strategies. The major/minor method is an old system [5] and uses 9 PRFs, the M of N method uses 8 PRFs, the result found by Simpson [7] [51 57 63 66 69 78 90 96], and the three evolutionary methods, with 5, 7 and 8 PRFs respectively. Although decodeability is not tested as part of the optimisation process, all the solutions used in Fig. 5 are fully decodeable in range and Doppler using any 3 PRIs out of each set. It is clear that the evolved solutions can far exceed the performance of the more traditional methods. The major/minor method with 9 PRFs can be outperformed by the new methods with only 5 PRFs, and the M of N method with 8 PRFs can be outperformed with only 7 PRFs.

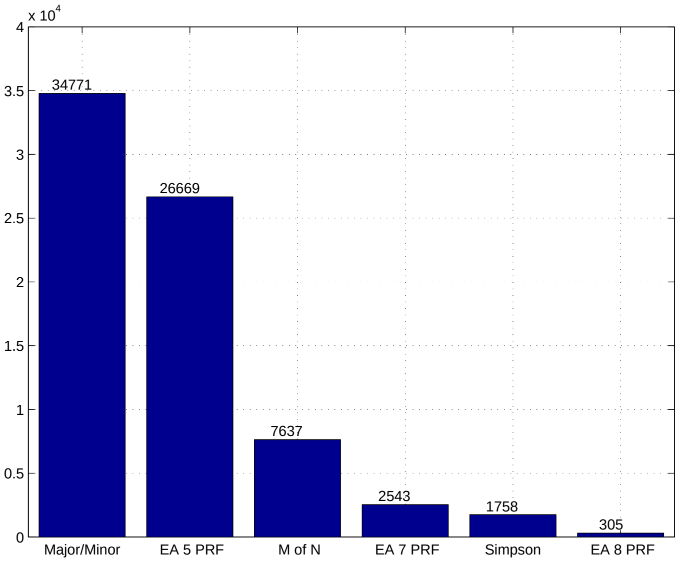

Fig. 5. Comparison of results found by different methods

VI. Conclusions

This work has shown that evolutionary algorithms are a feasible and effective alternative in finding optimum and near optimum solutions for the selection of PRF sets in medium PRF radars. Especially where the algorithm to calculate the blind zones is complex and therefore computationally intensive, undertaking exhaustive searches for optimum and near optimum solutions is a time and resource consuming procedure. The solutions found by the evolutionary algorithm are significantly better than those found by the main published methods of calculating the PRF set.

Evolutionary algorithms can have objectives that change with time and if the algorithm can be made to run fast enough, real time or near real time optimisation of the PRF set could be feasible. As many of the characteristics of the radar returns vary with such factors as altitude, terrain and platform velocity, operational performance gains could be made. A radar with adaptive PRF set selection would probably gain an advantage in terms of Electronic Surveillance Measures (ESM). A radar with a constantly changing waveform featuring a variable number of PRFs would undoubtedly present a more difficult problem to anyone attempting to deploy Electronic Counter Measures against it.

Existing PRF set selection methods can only cope where the solution space is small enough to search exhaustively, or where there are many constraints allowing only a few feasible solutions. Evolutionary Algorithms can find operationally sound solutions where the solution space is too big to search exhaustively. In the past, having to reduce the problem complexity has limited the design of medium PRF radars and it may be possible to design fundamentally more complex radars with far superior performance using the evolutionary techniques.

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